A UNIFIED APPROACH TO JOINT REGIONAL/TELESEISMIC CALIBRATION AND EVENT LOCATION WITH A 3D EARTH MODEL

William L. Rodi¹, Delaine T. Reiter², and Stephen C. Myers³

Massachusetts Institute of Technology¹, Weston Geophysical Corporation², and Lawrence Livermore National Laboratory³

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ABSTRACT

This newly initiated project will develop and test a methodology for locating seismic events from combined data sets of regional and teleseismic arrival times, based on consistent travel-time predictions from a unified 3D Earth model. One focus of the project is to address the practical difficulty of raytracing in 3D models, which has been a serious impediment to the pursuit of 3D event location methods. We will investigate whether, for teleseismic travel-time prediction, approximate techniques, in particular linearization around rays calculated in a 1D reference model, are adequate for the purpose of event location as they are commonly assumed to be for the purpose of global tomography. A second focus of the project is tomographic calibration of a 3D model with combined regional and teleseismic data from earthquakes and ground-truth events. We will investigate the hypothesis that a joint regional/teleseismic calibration will lead to a noticeable improvement in location accuracy over the modest and inconsistent improvements 3D models have yielded to date. Additionally, we will consider whether travel-time prediction errors inferred from a tomographic uncertainty analysis can provide an appropriate weighting of various teleseismic and regional phases to optimize location accuracy even further. We plan to validate our methodology with catalog data from Asia, the Middle East and North Africa, using available regional crust/upper mantle models for these areas in conjunction with published 3D global models of the deeper mantle. Special attention will be given to south-central Asia, where Weston Geophysical and MIT are currently applying body-wave and surface-wave tomography to develop a regional model of the crust and upper mantle. This project will attempt to improve and extend the Weston/MIT model by adding teleseismic constraints.
OBJECTIVE

The development of three-dimensional models of Earth structure continues to be an active field of seismology advancing along several fronts. A small sampling of these efforts include global travel-time tomography (e.g., van der Hilst et al., 1997; Antolik et al., 2003), global surface-wave dispersion tomography (e.g., Ritzwoller et al., 2002), regional models for Asia (Murphy et al., 2005; Li et al., 2006) and the Middle East (Pasyanos and Walter, 2002). An important question for nuclear monitoring is whether these, or improved, 3D Earth models can produce routine event locations with significantly reduced errors compared to 1D models when data at all event-station distances are used together.

The answer to this question would seem to be a definite yes when you consider that travel-time predictions from a 1D model like ak135 (Kennett et al., 1995) differ from observed travel times systematically by up to 10 seconds at regional distances and a few seconds at teleseismic distances, more than enough to induce event mislocations of tens of kilometers. An example from our own work on tomography and location supports this view. Weston Geophysical Corp. and MIT are developing a new 3D crust and upper mantle model for southern Asia (see Figure 1) based on Pn travel times from the EHB database (Engdahl et al., 1998) and Rayleigh-wave dispersion in the period range 10-150 sec (see Reiter and Rodi, 2008). The inversion model, which we call JWM, constrains the P and S velocities only at depths less than 410 km, so we are validating JWM by relocating ground-truth events with local and regional arrival-time data. The results of two such tests are shown in Table 1, where mislocations are listed for the 1998/05/05 nuclear test in Pakistan and the 1998/05/28 nuclear test in India. We see that, even with a large number of regional phases, the locations based on ak135 are in considerably more error than the locations derived from our 3D model. (These tests used only regional P arrivals (Pn) and the results shown are for constrained focal depth.)

Figure 1. Study region for the Weston/MIT regional tomography project, currently underway. The project is applying joint body-wave travel-time and surface-wave dispersion tomography to derive a self-consistent model for the P-wave and S-wave velocities of the crust and upper mantle. The inversion model (JWM) is described by Reiter and Rodi (2008).
Table 1: Location errors for the 1998 Pakistan and India nuclear tests, using regional arrival-time data.

<table>
<thead>
<tr>
<th>Event</th>
<th>Model</th>
<th>No. of data (for reg.)</th>
<th>Epicenter Error (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pakistan test</td>
<td>ak135</td>
<td>35</td>
<td>12.6</td>
</tr>
<tr>
<td>Pakistan test</td>
<td>JWM</td>
<td>35</td>
<td>2.9</td>
</tr>
<tr>
<td>India test</td>
<td>ak135</td>
<td>27</td>
<td>37.2</td>
</tr>
<tr>
<td>India test</td>
<td>JWM</td>
<td>27</td>
<td>16.3</td>
</tr>
</tbody>
</table>

The results in Table 1 are very encouraging since events of monitoring interest are typically small and yield mostly regional observations. However, such events often yield teleseismic observations, too, as in the case of the North Korea test on 2006/10/08. Furthermore, the location of large events is also important for nuclear monitoring since well-located events, of any size, serve as reference events for calibration and modeling studies. Therefore, we also relocated the explosions in Pakistan and India with the addition of the many teleseismic P-wave observations available, using stations up to 50° from the events. We still used JWM for predicting Pn travel times but, since our model is constrained only to 410 km in depth, we used ak135 for predicting teleseismic travel-times. We compared this mixed approach with purely ak135 solutions. The results are in Table 2.

Table 2: Location errors for the 1998 Pakistan and India nuclear tests, using regional and teleseismic data.

<table>
<thead>
<tr>
<th>Event</th>
<th>Model</th>
<th>No. of data</th>
<th>Epicenter Error (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pakistan test</td>
<td>ak135</td>
<td>35 Reg. 85 Tel.</td>
<td>13.0</td>
</tr>
<tr>
<td>Pakistan test</td>
<td>JWM</td>
<td>35 Reg. 85 Tel.</td>
<td>16.6</td>
</tr>
<tr>
<td>India test</td>
<td>ak135</td>
<td>27 Reg. 99 Tel.</td>
<td>15.2</td>
</tr>
<tr>
<td>India test</td>
<td>JWM</td>
<td>27 Reg. 99 Tel.</td>
<td>5.9</td>
</tr>
</tbody>
</table>

When ak135 is used for both regional and teleseismic times, the epicenter mislocation of the Pakistan test is about the same with or without teleseismic data (line 1 of Tables 1 and 2). However, when JWM is used for regional times and ak135 for teleseismic, the mislocation increases significantly (line 2 of the tables). This degradation does not happen in the case of the India explosion, however (lines 3 and 4 of the tables), for which the addition of teleseismic data reduces the mislocation with either regional model. Notably, JWM still performs better than ak135. The conclusion we draw is that mixing a calibrated regional model with ak135 (which is only calibrated in an average global sense) can make event locations worse, although not every time as we have seen.

Other studies have demonstrated improvement in regional event locations based on 3D crust/upper mantle models, although they have been restricted mostly to a priori, rather than tomographic, models (Ryaboy et al., 2001; Johnson and Vincent, 2002; Pasyanos et al., 2004; Flanagan et al., 2007). However, relatively little work has been on joint regional/teleseismic location with 3D models. In one of the few published studies, Yang et al. (2004) found that event locations obtained by combining regional and teleseismic data were only more accurate than locations found from the separate data sets if they corrected for a bias between the teleseismic and regional travel-time predictions, each made from a different 3D model (CUB1.0 for regional, J362 for teleseismic). The correction they inferred is small (0.79 sec), which is consistent with the fact that 3D crust/upper mantle effects on teleseismic travel times are relatively small (<2 sec). But event small systematic shifts in teleseismic travel times can induce large location errors owing to the high apparent velocity of teleseismic arrivals.

It is clear to us that the most promising approach to event location with teleseismic and regional observations is to computer travel times consistently from a single model. Further, that model should be jointly calibrated with both regional and teleseismic data.

**RESEARCH PLANNED**

This project will develop and test a self-consistent unified methodology for seismic event location and location calibration based on 3D Earth models. Some of the questions we will address are the following:
Given a unified 3D model of the Earth, what numerical algorithms are best suited to the forward modeling of travel times for various seismic phases for use in event location? The phases we will be concerned with are the local, regional and teleseismic phases that are routinely available in earthquake catalogs such as the EHB database, and those potentially observable in monitoring applications.

Using observations of the same phases from reference events and earthquake catalogs, or from temporary station deployments, what is the best strategy for calibrating the unified Earth model? An important issue here is whether regional models of the crust and upper mantle can be calibrated separately from lower mantle models, or whether a joint regional/teleseismic tomography is essential. Related to this, can a tomographic inversion for the crust and upper mantle include constraints from teleseismic data without varying the deeper mantle structure?

Can a rigorous approach to model uncertainty provide automatic data weights for an event location algorithm, such that well-calibrated and weakly or un-calibrated travel-time predictions are combined optimally? Location algorithms used in nuclear monitoring currently employ estimates of “model errors” to perform this weighting. We will investigate whether more formal uncertainty estimates derived from tomographic calibration, including covariances between travel-time predictions along similar paths, can perform better.

We plan to address these questions using a variety of available 3D Earth models, including the JWM model referred to above, the WENA1.0 model developed at LLNL (Pasyanos et al., 2004), and global mantle models such as J362D28 (Antolik et al., 2003) and MITP07 (Li et al., 2008).

Statement of the Problem

The forward model in seismic event location employs a set of travel-time functions, each predicting the travel time of a seismic phase from any potential event hypocenter to an observing station. This prediction is based on wave propagation through a reference velocity model. Since the Earth's true velocity differs from the reference model, the forward model admits a prediction error (Billings et al., 1994).

Let \( v_0 \) denote the velocity of the reference model (focusing on just the velocity for \( P \) waves), and let \( v_E \) be the Earth's true velocity. We can express the prediction error as

\[
e(x) = T(x; v_E) - T(x; v_0),
\]

where the travel-time function, \( T(x; v) \), integrates \( 1/v \) along the raypath connecting the station to the hypocenter \( x \). Prediction errors add to the observational errors in arrival-time picks, and a difficult but important problem is to account for both kinds of error in locating an event.

Calibration is the task of reducing or correcting the prediction errors in travel times. One calibration approach that has been used in nuclear monitoring applications (e.g., Myers and Schultz, 2000) is the kriging, or geostatistical interpolation, of observed travel-time residuals from ground-truth events, which leads to a correction function or surface, \( \Delta T(x) \), for a given station/phase. This method works well when all of the network stations have observed a GT event (or events) near an event of interest. The forward model used in event location is then taken as \( T(x; v_0) + \Delta T(x) \). Model-based calibration, in contrast, attempts to correct the velocity model itself: replacing \( v_0 \) with an improved model \( v_C \equiv v_0 + \Delta v \). The corrected model can be derived with travel-time tomography, applied to data similar to that used in kriging, but also can incorporate a variety of other constraints. An advantage of model-based calibration is that it allows the calibration of stations and seismic phases which have not been directly observed, as long as \( v_C \) is defined for relevant portions of the Earth and, additionally, appropriate ray tracing techniques are available for the phases of interest.

The conceptual goal of this project can now be stated as follows. Given a globally defined 3D model, \( v_C \), we wish to identify forward modeling and event location techniques which allow us to take advantage of all the travel-time data available for locating an event. Conversely, we wish to develop tomographic methods for calibrating \( v_C \), given the available travel-time observations from ground-truth and other past events.
3D Raytracing

A critical choice in both event location and travel-time tomography with 3D Earth models is the choice of numerical technique to use for calculating travel times between sources and receivers. The choice involves somewhat different issues in location and tomography. First, an event locator requires travel-time calculations for arbitrary hypocenters as it searches for a best-fitting event location, while tomography is usually performed with event locations held fixed. Second, tomography needs the sensitivity of each calculated travel time to the velocity structure of the Earth, i.e. the ray trajectory connecting source and receiver, or its equivalent.

For travel-time calculation in a 1D reference model (e.g., *ak135*) the choice is straightforward since analytical solutions for travel times and rays, for any seismic phase (direct, refracted, diffracted or converted) are available and easily implemented in fast algorithms (Buland and Chapman, 1983). In contrast, travel-time calculation in 3D models presents many challenges.

Global travel-time tomography has generally been performed with a linearized approximation to the travel-time function. In this approximation, the slowness of the 3D model (1/vC) is integrated along the raypaths computed in the reference model, v0, with the reference model taken as 1D. We can identify the ray connecting a given station and a given event hypocenter, xev, with a sensitivity kernel, a(x,xev;v0), that is concentrated along the ray trajectory. Then the linearized approximation to the travel time through vC can be written as

\[ T(x_{ev},v_C) \approx T(x_{ev},v_0) + \int dx \ a(x,x_{ev};v_0) \left[ v_C^{-1}(x) - v_0^{-1}(x) \right] \]

(2)

We point out that this equation also applies to finite-frequency travel times, which Li et al. (2006) and others have used to model long-period body-wave observations. In this case, a(x,xev;v0) becomes a “banana-doughnut” sensitivity kernel (e.g., Dahlen et al., 2000) evaluated for the reference velocity model.

For the calculation of local and regional direct P arrivals, linearization is not adequate, as we will see in a moment. For this reason, Reiter and Rodi (2006) use the finite-difference ray tracing method due to Podvin and Lecomte (1991), as implemented in software by Lomax (2000). Reiter et al. (2005) augmented the Podvin-Lecomte (P-L) algorithm with a back-chaining algorithm to generate sensitivities of travel times with respect to the velocity parameters of the model. Since the P-L algorithm is written for flat-Earth models in Cartesian coordinates, we have integrated it into our tomography software with appropriate transformations between model parameterizations and sensitivities. Reiter et al. (2005) provide additional details of the ray tracing techniques we use for regional tomography.

We illustrate the importance of 3D ray tracing in modeling regional travel times using our JWM tomographic model. First, we compare travel times calculated through JWM, using the Podvin-Lecomte algorithm, to the travel times calculated through ak135. In each case, the times are calculated for the 76,000 event-station pairs in our tomography data set (extracted from the EHB catalog). The difference between the JWM and ak135 times for these paths is plotted as a function of epicentral distance in the left panel of Figure 2. Not surprisingly, the models predict Pn times differing by several seconds.

Can Pn travel times through JWM be approximated with linearization around ak135, using Equation (2)? The right panel of Figure 2 shows the difference between the P-L calculated times (“Nonlin.”) and the ak135 linearized approximation (“Linear”). We see that the errors in the linearized times are as high as 25 sec. Much of this difference owes to the thick crust over much of the model region, exceeding 75 km thickness beneath the Himalayas. Therefore, large pieces of the ak135 Pn raypaths are incorrectly located within the JWM crust. While it might be possible to correct for this effect, or optimize the 1D model used to obtain the reference raypaths, the fact is that integrating a 3D velocity model along reference rays from a 1D model cannot accurately model Pn travel times through a complex, 3D Earth model.
Figure 2. Experiments with regional travel-time calculation performed with the 3D tomographic model JWM of Reiter and Rodi (2006). Left: Difference between calculated travel times for JWM and ak135. The JWM times were calculated with the Podvin-Lecomte (P-L) finite-difference algorithm. Right: Difference between calculated travel times from JWM, obtained two ways: “nonlinear” times obtained by the P-L method, and “linearized” times obtained by integrating the JWM slowness over ak135 raypaths. In both panels, times are plotted for the 76,000 Pn paths used in the JWM tomography.

However, it is interesting that the linearized travel times are not too bad for the longest Pn paths, i.e., beyond 14°, corresponding to rays bottoming between 210 and 410 km in the ak135 model. This is because the crustal thickness variations affect these raypaths much less than for shorter epicentral distance, making Equation (2) more accurate. Also, the velocity differences between JWM and ak135 in the mantle are smaller than in the crust.

Therefore, it is reasonable to ask whether the linearization approach would even more accurately predict travel times in the teleseismic distance range. This is a question we will investigate in detail as part of the proposed project. A preliminary look into the matter is offered in Figure 3. This figure is analogous to Figure 2, but the travel-time calculations were performed only for a line of stations emanating from station JYP (33°N, 75°E) along a great circle at an azimuth of -30° from north, out to near-teleseismic distances. To calculate teleseismic times in JWM with Podvin-Lecomte, JWM was extended below 410 km depth with the ak135 velocity profile. The left panel of Figure 3 shows that the influence of the 3D structure in JWM on teleseismic times is less than 1 second, given this extrapolation at depth. The panel on the right, nonetheless, shows that the teleseismic travel times through JWM are accurately modeled by the linearized approach applied with ak135 raypaths. It remains to be seen whether the presence of deep 3D structure alters this conclusion.

Figure 3. Left: Difference between calculated travel times from JWM and ak135. Right: Calculation error of travel times from JWM by using linearization.
Event Location with a 3D Model

The location validation effort in the Weston/MIT tomography project for southern Asia is using the grid-search location algorithm known as GMEL (Grid-search Multiple-Event Location), developed at MIT under previous projects and extended to 3D models in a previous joint project with Weston Geophysical Corp. GMEL uses travel-time predictions for either 1D reference velocity models or 3D reference models, using interpolation of pre-calculated tables in each case. A summary description of the numerical techniques used in GMEL is available in Rodi (2006). We plan to use GMEL in the proposed work, but we will investigate whether teleseismic travel times for 1D models and linearized travel times for 3D models can be efficiently calculated during the grid-search location process (“on-the-fly”). This will address an important implementation issue in the use of 3D models for event location and, if on-the-fly calculations are feasible, GMEL will be a more useful tool for the project.

Another previous effort we plan to take advantage of in the proposed work is a concurrent project on covariance modeling. Rodi and Myers (2006) presented a method for computing variances of, and covariances between, travel-time predictions from a reference model, based on a geostatistical description of the velocity anomalies in the Earth relative to the reference model. The numerical technique for doing this integrates travel-time sensitivities with a velocity model covariance function, \( C(x,x') \), that characterizes the velocity heterogeneity. Thus, the covariance between the travel-time predictions along two paths, \( i \) and \( j \), is given by

\[
\sigma_{ij}(x_{ev}) = \int dx \int dx' a_i(x,x_{ev};v_0) a_j(x,x_{ev};v_0) C(x,x'),
\]  

(3)

where \( a_i \) and \( a_j \) are the sensitivity kernels for the two paths. An example of this calculation, taken from Rodi and Myers (2006), is shown in Figure 4.

We plan to implement the use of travel-time covariances in GMEL, and to extend the technique by replacing \( C(x,x') \) with the \textit{a posteriori} covariance that results from performing travel-time tomography. This capability will accomplish a key part of our unified approach, with the data in an event location automatically and optimally weighted in correspondence to how well their travel-time predictions are calibrated.

CONCLUSIONS AND RECOMMENDATIONS

Replacing a 1D Earth model with a global 3D Earth model for routine event location holds the promise of achieving much greater location accuracy, but it is an enormous undertaking when one considers all of the steps involved: model integration, calibration and validation, and efficient implementation for use in a locator. Moreover, it is an undertaking that is highly vulnerable to the one-step-forward, two-steps-back syndrome, with improvement on one front (e.g., better regional travel-time predictions) offset by regression on another (e.g., discrepancy between regional models, inconsistency with teleseismic predictions). The greatest benefit of our project will be a strategy for achieving the maximum effect from future efforts to develop and implement 3D Earth models for event location. Additionally we expect this project to have some concrete benefits beyond the methodology it develops. These include an improvement to the Weston/MIT model of the crust and upper mantle in southern Asia, incorporating teleseismic constraints, and an improved version of the EHB catalog for southern Asia.
Figure 4. **Top** - Diagram of the network geometry used for testing our numerical approach to modeling travel-time covariances among a set of stations. There are 12 stations at regional distances from an earthquake in northeastern Iran. **Bottom Left** - The standard deviation of the travel-time prediction error at each of the 12 stations, plotted as a function of epicentral distance. **Bottom Right** - Correlation coefficient between station pairs, plotted versus inter-station azimuth. The calculations assumed that velocity variations in the Earth have a standard deviation of 2% and correlation lengths of 300 km laterally and 100 km vertically.

**REFERENCES**


Reiter, D., and W. Rodi (2008). A new regional model 3-D velocity model for Asia from the joint inversion of P-wave travel times and surface-wave dispersion data, these Proceedings.


