RESOLVING INFRASOUND SIGNAL BACK AZIMUTHS WITH ARRAYS OF OPTICAL FIBER SENSORS

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ABSTRACT

Optical Fiber Infrasound Sensors (OFIS) are long compliant tubes wrapped with two optical fibers that interferometrically measure the differential pressure variation along the length of the tube. Because each sensor averages spatially along the path of the tube, the frequency response of the recorded pressure variation is a function of the orientation of the OFIS sensor relative to the back azimuth and incidence angle of the incoming wave. We have exploited this property to investigate the ability of various OFIS array geometries to determine the back azimuth of infrasound signals. We have found that an array comprised of two orthogonal 89-m-long OFIS having their centers separated by 63 m can resolve the back azimuth of most arrivals having good signal-to-noise ratio. We find a good match between the back azimuths determined from our technique and those determined from the same signals recorded on the co-located pipe array I57US using the Progressive Multichannel Cross-Correlation technique. Based on our results, we are proposing to build an OFIS array that will be able to resolve signals from all directions and with smaller signal-to-noise ratios.
OBJECTIVES

Optical Fiber Infrasound Sensors (OFIS) are a unique way to detect acoustic signals, in particular, signals in the infrasound band (below 20 Hz), which propagate approximately in the horizontal plane. We defer a detailed instrument description to Zumberge et al. (2003). In general, a 1310-nm laser illuminates a single-mode fiber, which is split into two (Figure 1). A lock-in amplifier is then used to generate a $\pi/2$ phase lag in one of the fibers while modulating the length difference by a small fraction of a wavelength at several hundred kHz with a piezoelectric crystal (PZT Modulator). The two fibers are then helically wrapped around a sealed, compliant tube, one with twice as many wraps as the other. At the end, the two fibers are recombined and yield an interference pattern that is recorded by a photo detector. For an ambient pressure variation, the tube expands and/or contracts, changing the difference in the optical-fiber path length each laser must travel. This difference is measured by the number of fringes counted in the interference pattern, and related to pressure by an empirically determined calibration factor of 2.52 Pa m rad$^{-1}$.

An OFIS has several advantages over other infrasound sensors such as pipe and hose arrays. It is relatively inexpensive to build, deploy, and maintain. It is also easy to deploy, and requires much less space, making it highly portable. There is also evidence that in low-wind conditions, the noise floor of the OFIS is lower than the pipe and hose arrays for frequencies down to 1 Hz. However, pipe and hose arrays are more intuitive, and have led to the development of various techniques for determining the signal characteristics (back azimuth and incidence angle), characteristics that are important for determining the source and location of infrasound signals. Finally, the OFIS appears to be highly responsive to high frequencies (up to at least 100 Hz), which makes it ideal for detecting signals in both the infrasound and low-frequency audible sound band.

The objectives of this research are to (1) determine the number and configuration of OFIS that is necessary to resolve the back azimuth of infrasound signals, (2) develop a generic technique for estimating back azimuths using an $n$-OFIS array, and (3) test the technique by comparing back azimuths estimated from synthetic signals and real signals recorded at Piñon Flat Observatory (PFO, I57US) in the southern California high desert.

Figure 1. Schematic of the Optical Fiber Infrasound Sensor (OFIS) design that is being implemented at PFO (I57US).
RESEARCH ACCOMPLISHED

Sensor Directivity

The response, $R$, of an OFIS relative to a point detector is a function of the orientation of the signal propagation with respect to the length of the OFIS:

$$R = \left| \text{sinc} \left( \frac{L \pi f}{V_a} \cos(\theta) \right) \right|$$  \hspace{1cm} (1)

where $f$ is frequency, $L$ is the length of the OFIS, $V_a$ is the sound speed at the Earth’s surface, and $\theta$ is the angle between the incident ray path and the OFIS

$$\theta = \cos^{-1} (\cos(\theta_b) \sin(\theta_i))$$  \hspace{1cm} (2)

where $\theta_b$ is the back azimuth, and $\theta_i$ is the incidence angle. For typical infrasound signals, $\theta_i$ is approximately 90° (horizontal), and $\theta = \theta_b$. Figure 2a shows a polar plot of $R$ as a function of angle from the long axis of the OFIS for three typical infrasound signal frequencies.

![Figure 2a](image)

**Figure 2.** Frequency response $R$ for an 89-m long OFIS as a function of frequency and angle $\theta$ (equations 1-2).

The response $R$ varies remarkably as a function of frequency (Figure 2b). Therefore, if the back azimuth for a signal with a significant bandwidth is known, one can deconvolve $R$ from the recorded signal $S_r$ to determine the actual signal waveform $S_w = S_r / R$.

Two-OFIS Array

We exploit the dependence of the frequency response on the back azimuth ($\theta_b$) and test the capabilities of a two-OFIS array in resolving $\theta_b$ for synthetic and real infrasound signals recorded at PFO.

Algorithm

The recorded OFIS signal $S_r = f(S_w, \theta, L)$, where only $S_w$ and $\theta$ are the unknowns. One can therefore estimate $S_w$ and $\theta$ if one records the signal on two OFIS with different orientations (Figure 3)

$$S_{r1} = f(S_w, \theta_1, L)$$  \hspace{1cm} (3)

$$S_{r2} = f(S_w, \theta_2, L)$$
where \( \theta_1 \) and \( \theta_2 \) are related by the array configuration. We do this by substitution, i.e., using \( S_{r_1} \) to predict \( S_w^p \), and then \( S_{r_2}^p \)

\[
S_{r_2}^p = S_w^p R_2 = S_{r_1} R_2 / R_1
\]  

(4)

We make the assumption that the incidence angle \( \theta_i = 90^\circ \), and perform a grid search over trial \( \theta_b \) (between 0-90\(^\circ\) from the OFIS1 azimuth in 1\(^\circ\) increments) to minimize the L2 misfit between \( S_{r_2} \) and \( S_{r_2}^p \). We currently perform the deconvolution in the frequency domain by using the water-level technique to avoid numerical instability, i.e., we raise all near-zero amplitudes in the denominator frequency spectrum to a “water level” of 1% of the maximum amplitude (e.g., Langston, 1979). We also calculate formal 2\( \sigma \) error bars using the technique of Silver and Chan (1991), which assumes the misfit function is the sum of squares of a random chi-squared noise process.

**Figure 3. Layout of the two-OFIS array with centers separated by 63 m.**

Because an OFIS integrates the change in pressure, a scalar field, along the length of the tube, one cannot determine the quadrant from which a signal originates (Figure 3). Any angle from OFIS1 is part of a complementary set of four angles from which an incoming signal has identically recorded shapes on both OFIS. To get around this ambiguity, we exploit the fact that the two OFIS are separated by 63 m, and perform a cross-correlation during each trial \( \theta_b \) to find the optimum time separation between \( S_{r_2} \) and \( S_{r_2}^p \) before calculating the misfit. Then for the optimum \( \theta_b \), we check the corresponding \( dt \), which determines from which of the four angles it originated assuming \( \theta_i = 45-90^\circ \). The resulting misfit function contains misfits from the optimally time-shifted \( S_{r_2} \) and \( S_{r_2}^p \). Some improvement would come from incorporating the predicted \( dt \) from each trial \( \theta_b \) to time shift the signals before the misfit is calculated. This should only require a slight increase in the computation time required for the calculation since the number of Fourier transforms does not change, but it may increase the ratio between the maximum and minimum misfit for most signals.

The angle halfway between both OFIS is an angle for which the recorded signals on both OFIS should be identical (Figure 3). We term this special angle the angle of ambiguity because one cannot resolve between it, its 180\(^\circ\) complement, or a signal with a low incidence angle.

This two-OFIS technique is completely generic, and could be applied to an \( n \)-OFIS array. However, such an approach requires the deconvolution of \( n \) functions with holes in the amplitude spectra. If water-level deconvolution is performed, it invariably introduces noise into the resulting waveforms, the magnitude of this noise depending on the recorded signal bandwidths. We believe the above technique could be made more stable by using all recorded OFIS signals to estimate the predicted waveform signal for each trial \( \theta_b \) and \( \theta_i \), then predict the recorded signals for each OFIS

\[
S_{r_k}^p = S_w^p R_k = \frac{\sum_{i=1}^{n} S_{r_i}}{\sum_{j} R_j} R_k
\]  

(5)
where $S_{rk}^p$ is the predicted recorded signal for the kth OFIS and n is the number of OFIS in the array. The larger n is, the greater the chance of resolving the properties of signals with a low signal-to-noise ratio.

**Testing the Two-OFIS Algorithm on Synthetic Data**

We generated synthetic data based on the infrasound signal from a Vandenberg Delta II rocket launch (8 September 2001), which was recorded by an OFIS oriented perpendicular to the signal back azimuth ($\theta_b$) at PFO. We use equations 3a-b to calculate what would be recorded by the OFIS sensors for $\theta_b=290^\circ$ and $\theta_i=90^\circ$, time shift the signals appropriately, add white noise, frequency filter the signals between 0.2-5 Hz, and apply the grid-search technique described above to these waveforms. Figure 4 shows the result of this experiment for noise levels ranging linearly between 0.01-100% as determined on $S_{r1}$. The misfit as a function of trial $\theta_b$ is shown at the top, and the waveforms for $S_{r2}$ (recorded) and $S_{r2}^p$ (optimum) are shown below.

![Figure 4. Testing technique with synthetic data for an assumed 290° back azimuth (BAZ).](image)
We find that the algorithm approximately identifies the input $\theta_b$ for noise levels up to $\sim 10\%$, which leads to correlation coefficients (qual) between $S_{r_2}$ and $S_{r_2}^p$ of 0.75-1.0. As the noise level increases, the global minimum (indicated by green vertical lines) moves progressively toward the azimuth of ambiguity, and then toward the azimuth of OFIS2. This behavior is not surprising. As the noise increases, the signal-to-noise ratio (SNR) on $S_{r_1}$ decreases much faster than that on $S_{r_2}$ (see Figure 2b). At trial $\theta_b$ near the azimuth of OFIS1, the noise is being amplified because $R_1$ is only flat for low frequencies (equation. 4), which results in a very noisy $S_{r_2}^p$ and $S_{r_2}^p$. As trial $\theta_b$ goes to the azimuth of OFIS2, the predicted $S_{r_2}^p$ gets smaller in amplitude because $R_2$ gets flatter. The subsequent convolution with $R_2$ yields a much lower amplitude $S_{r_2}^p$. So the misfit-curve trend at high noise levels simply represents the decrease in $S_{r_2}^p$ RMS amplitude, and has nothing to do with the coherence between $S_{r_2}$ and $S_{r_2}^p$. The toward-OFIS2 global-minimum trend represents the gradual loss of signal resolution and reversion to this state. The same basic conclusion is reached for different signal bandwidths and deconvolution water-level parameters.

We also find that our formal error bars are too optimistic, and do not accurately account for the deviation between the optimum and actual $\theta_b$. With a multi-element OFIS array however, one could perform a bootstrap procedure to determine the error bars, although this would make the technique much more computationally intensive. Error bars could also be crudely estimated by performing the technique along a running window across the signal, and measuring the standard deviation of all the $\theta_b$ estimates at the end (this is what the PMCC code does).

The most important conclusion of the synthetic tests is that the algorithm should work for real signals (see next section) with good SNR. We also tested this method by performing a 2-D grid search over $\theta_b$ and $\theta_i$, and found that one can also determine $\theta_i$ for $\theta_i = 45-90^\circ$ if $\theta_b$ is a significant distance from the azimuth of ambiguity (the distance depending on the SNR). But we focus on the assumption that $\theta_i = 90^\circ$ because most infrasound signals of interest obey this assumption and the 1-D grid search is computationally less intensive by about two orders of magnitude.

**Testing the Algorithm on Real Data**

We also tested the method on 93 real infrasound signals of good signal-to-noise ratio recorded on a two-OFIS array during the first quarter of 2004 at PFO. To determine if the estimated OFIS $\theta_b$ are accurate and to distinguish between horizontally traveling and near-vertically traveling infrasound signals, using the Progressive Multichannel Cross-Correlation algorithm (Cansi, 1995), we also analyzed the same signals that were recorded by the co-located IMS pipe-array consisting of four 18-m and four 70-m diameter rosettes spread over a 1,400 m region (Hedlin et al., 2003).

Figure 5 compares $\theta_b$ calculated from both methods, and shows the residual deviation after the error bars are used to explain as much of the misfit as possible. We find that for most of the signals, there is a very good correlation (Figure 5b). For the other signals, there is a significant deviation of up to $30^\circ$ and greater than $30^\circ$ for $\theta_b = 105-125^\circ$, 215-225$^\circ$, and 280-330$^\circ$. These OFIS estimates deviate from the PMCC estimates in the counterclockwise direction. It may be significant that the first and last problem region are $\sim 180^\circ$ apart, and the deviation of the OFIS $\theta_b$ is toward OFIS2, suggesting noise may be the responsible for the deviation (see Figure 4). If this is the case, the middle problem region, which is only $10^\circ$ wide, originates from a different source.

We used several methods to determine the orientation of each OFIS to within $0.3^\circ$. We also performed an analysis on the pipe-array orientation by using PMCC to analyze the signals of 16 known regional mine-blast events from $\theta_b = 280-340^\circ$, and found that the theoretical $\theta_b$ matched those from PMCC within $10^\circ$. Ignoring the possibility that there are $\theta_b$ (and $\theta_i$) outside 280-340$^\circ$ for which PMCC does not yield reliable estimates, it tentatively appears that the above technique has a difficult time resolving the correct $\theta_b$ in those problem regions.
Figure 5. Comparison of back azimuths calculated from signals recorded on a two-OFIS array using the technique described in this paper and a more traditional pipe array using the PMCC code.
CONCLUSIONS AND RECOMMENDATIONS

Synthetic and recorded infrasound signals at PFO (I57US) suggest that a two-OFIS array with centers separated by 63 m is generally capable of resolving the back azimuth of signals that have a good signal-to-noise ratio with the generic technique we present above (Figures 4 and 5). Based on these encouraging results, we suggest that the above technique, when combined with a few enhancements and applied to a multi-element OFIS array, should be capable of resolving the properties of signals with various signal-to-noise ratios from all possible orientations. Specifically, we are proposing to design a six-arm OFIS array oriented in a radial pattern with each OFIS separated by an angle of $60^\circ$ (although a three-arm OFIS array is enough to alleviate the problems associated with the azimuth of ambiguity, the six-arm OFIS would provide redundancy, which is important when analyzing signals of low signal-to-noise ratio). Enhancements to the generic technique we present above include: (1) using the predicted $dt$ during the grid search rather than at the end, (2) using all OFIS signals for a trial $\theta_b$ and $\theta_i$ to directly calculate $S_w$, which avoids numerical instability (see equation 5), and/or (3) using a better deconvolution technique in lieu of the water-level method.

These results are significant because they indicate that an OFIS array, acting as a directional antenna, is capable of providing the same directional information as an array of conventional rosette filters while covering much less space. Conventional arrays rely on time delays between elements to determine $\theta_b$ and $\theta_i$, requiring significant separation between elements (hundreds of meters) to produce time differences. The OFIS reliance on directional frequency response should provide the same information within a more localized area -- a significant advantage when space is at a premium (e.g., on island sites).

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REFERENCES


